The second test for this course will be given in class on Friday, November 13. It covers the course from the first test through November 4. This includes Chapter Three and material from Chapter Four on determinants,. We only covered parts of Chapter Four. See the list of topics below for more detailed information about exactly what is included on the test. A few things are deliberately left out, such as the effect of change of basis on a homomorphism, Laplace's expansion, and affine transformations.

Note that even though Chapters One and Two will not be covered explicitly on the test, you might still need background material from those chapters, including row operations, echelon form, leading and free variables, linear combination, vector space, vector spaces of polynomials, linear independence, span, basis, and dimension.

The format of the test will be similar to the first test: four pages with definition and other short essay questions, longer essay questions, computational problems, and maybe some short proofs. There will probably be at least one longer, summary essay question. There will be no long or complex calculations on the test; if you think a problem requires a long calculation, you are missing a simple way to do it!

Here are some terms and ideas that you should be familiar with for the test:

one-to-one function; onto function; bijective function; inverse of a bijective function

linearity of a function $f\colon V \to W$: $f(r\cdot\vec{v}) = r\cdot\vec{v}$ and $f(\vec{v}+\vec{u}) = f(\vec{v}) + f(\vec{u})$

homomorphism, also known as linear map

Theorem: if $h\colon V \to W$ is a homomorphism, then $h\big(\vec{0}_V\big) = \vec{0}_W$

isomorphism (bijective homomrphism)

automorphism (isomorphism of a vector space with itself)

Theorem: if $h\colon V \to W$ is an isomorphism, then $h^{-1}\colon W \to V$ is also an isomorphism

every n-dimensional vector space is isomorphic to $\mathbb{R}^n$

a homomorphism is completely determined by its values on the elements of a basis

$\mathscr{R}(h)$, the range space of a homomorphism h

$\mathscr{N}(h)$, the null space (or kernel) of a homomorphism h

rank of a homomorphism (dimension of the range space)

nullity of a homomorphism (dimension of the null space)

Theorem: a homomorphism h is one-to one if and only if $\mathscr{N}(h) = \{\vec{0}\}$

$M_{m\times n}$, the vector space of $m \times n$ matrices

product $A\vec{v}$ of an $m \times n$ matrix A times a column vector $\vec{v} \in \mathbb{R}^n$

$A\vec{e}_i$ is the i^{th} column of A, where $\vec{e}_i$ is the standard basis vector

product AB of an $m \times k$ matrix A with a $k \times n$ matrix B

matrix multiplication is associative

if A is an $m \times n$ matrix, the function $h(\vec{v}) = A\vec{v}$ is homomorphism from $\mathbb{R}^n$ to $\mathbb{R}^m$

every homomorphism from $\mathbb{R}^n$ to $\mathbb{R}^m$ is given by multiplication by an $m \times n$ matrix

the product AB of two matrices represents the composition of the corresponding homomophisms

the $n \times n$ identity matrix I_n

the inverse A^{-1} of a non-singular $n \times n$ matrix

how to compute an inverse matrix using row reduction

diagonal matrix

elementary matrices, representing elementary row operations

homomorphisms V to W correspond to matrices, but only after choice of bases for V and W

$\text{Rep}_B(\vec{v})$, the representation of a vector $\vec{v} \in V$ in a basis B of V

$\text{Rep}_{B,D}(h)$, the representation of a homomorphism $h\colon V \to W$, where B, D are bases of V, W

writing $h(\vec{\beta}_i) = c_{1i}\vec{\delta}_1 + c_{2i}\vec{\delta}_2 + \cdots + c_{mi}\vec{\delta}_m$ to get the i^{th} column of $\text{Rep}_{B,D}(h)$

$\text{Rep}_{B,D}(h) \cdot \text{Rep}_B(\vec{v}) = \text{Rep}_D(h(\vec{v}))$

determinants

the determinant of a 2×2 matrix: $\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

properties of the determinant for $n \times n$ matrices: multilinear, antisymmetric, $|I_n| = 1$

an $n \times n$ matrix A is non-singular if and only if $\det(A) = 0$

effect of row operations on the determinant of a matrix

using row reduction to compute a determinant

determinant of a matrix product: $|AB| = |A| \cdot |B|$

determinant of an inverse matrix: $|A^{-1}| = \frac{1}{|A|}$