*This homework is due by **noon** on Tuesday, October 20*

***Note: Only problems 4, 5, and 6 are eligible for rewriting.***

**Problem 1.** Some of the following matrix products are not defined. For each product, you should either compute the product, if it is defined, or state why it is not defined.

**a)** $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 3 & -1 \\ -2 & 2 \\ 1 & 0 \end{pmatrix}$ **b)** $\begin{pmatrix} 0 & 3 & 5 \\ 2 & 1 & 7 \\ 1 & 3 & 0 \end{pmatrix} \begin{pmatrix} 5 & 2 \\ 3 & 1 \end{pmatrix}$

**c)** $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ **d)** $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ **e)** $\begin{pmatrix} 3 \\ 5 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 1 & 7 \end{pmatrix}$

**Problem 2.** Let $A = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix}$. Note that $A \in M_{4\times 1}$ and $B \in M_{1\times 4}$. Compute the matrix products $AB$ and $BA$.

**Problem 3.** Find the inverse of each of the following matrices, or show that the matrix has no inverse. For part (c), you should find the inverse by putting the augmented matrix

$$\left(\begin{array}{ccc|ccc} -1 & 3 & 0 & 1 & 0 & 0 \\ 2 & -1 & 5 & 0 & 1 & 0 \\ 1 & 2 & -5 & 0 & 0 & 1 \end{array}\right)$$

into reduced echelon form.

**a)** $\begin{pmatrix} 3 & -2 \\ 5 & 4 \end{pmatrix}$ **b)** $\begin{pmatrix} 6 & -4 \\ -3 & 2 \end{pmatrix}$ **c)** $\begin{pmatrix} -1 & 3 & 0 \\ 2 & -1 & 5 \\ 1 & 2 & -5 \end{pmatrix}$

**Problem 4.** We can define the derivative for all polynomials as a homomorphism $D\colon \mathscr{P} \to \mathscr{P}$. The null space $\mathscr{N}(D)$ is the set of all constant polynomials. We can form the composition $D \circ D$, which also maps $\mathscr{P}$ to $\mathscr{P}$. For a polynomial $q(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$, what is $D \circ D(q(x))$? What mathematical operation does $D \circ D$ compute? What is the null space of $D \circ D$?

**Problem 5.** Suppose that $V$ and $W$ are vector spaces and that $f\colon V \to W$ is a homomorphism. Suppose that $S \subseteq V$ and that $S$ spans $V$. Show that the set $f(S)$ spans the range space $\mathscr{R}(f)$ of $f$. (Note: $\mathscr{R}(f) = \{f(\vec{v}) \,|\, \vec{v} \in V\}$, and $f(S) = \{f(\vec{v}) \,|\, \vec{v} \in S\}$.)

**Problem 6.** Suppose that $V$ and $W$ are vector spaces and that $f\colon V \to W$ is a homomorphism. Assume that $h$ is one-to-one (which implies that the null space $\mathscr{N}(h)$ contains only $\vec{0}_V$). And suppose that $\langle \vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k \rangle$ is a linearly independent sequence of vectors in $V$. Show that the sequence $\langle f(\vec{v}_1), f(\vec{v}_2), \ldots, f(\vec{v}_k) \rangle$ is linearly independent (in $W$).