This homework is due by the end of the day on Wednesday, Sept. 16.
Be sure to show your work and explain your reasoning!

Problem 1. Produce **three** other matrices that are row equivalent to the following matrix. State what you did to get each new matrix. If you understand what row equivalence means, this exercise is very easy!

$$\begin{pmatrix} 3 & 4 & -2 \\ 1 & 1 & 7 \\ -5 & 3 & 1 \end{pmatrix}$$

Problem 2. Apply row operations to put the matrix on the left below into reduced echelon form. Show the full sequence of operations that you apply. The only correct answer is shown on the right. The point of the problem is to show the computation.

$$\begin{pmatrix} 1 & 0 & -2 & 0 & 2 \\ 0 & 1 & 3 & 2 & 7 \\ 1 & 2 & 4 & 2 & 8 \\ -2 & 2 & 10 & 2 & 2 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & -2 & 0 & 2 \\ 0 & 1 & 3 & 0 & -1 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Problem 3. Assuming that the matrix in the previous problem represents a **homogeneous** system of linear equations, use the reduced echelon form of the matrix to find the solution set of that system. (The constant terms in the equations, which are all zero, are omitted from the matrix!)

Problem 4. Put the following eight matrices into groups, so that each matrix is row-equivalent to all the other matrices in the group, but not row-equivalent to matrices from other groups. (Remember how to use reduced echelon form to tell whether two matrices are row equivalent.)

a) $\begin{pmatrix} -1 & 3 \\ 2 & -6 \end{pmatrix}$ **b)** $\begin{pmatrix} 2 & 4 \\ 1 & 5 \end{pmatrix}$ **c)** $\begin{pmatrix} 0 & 1 \\ 0 & 3 \end{pmatrix}$ **d)** $\begin{pmatrix} 3 & 6 \\ -2 & -4 \end{pmatrix}$

e) $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ **f)** $\begin{pmatrix} \frac{1}{3} & -1 \\ -3 & 9 \end{pmatrix}$ **g)** $\begin{pmatrix} 0 & 0 \\ 1 & 2 \end{pmatrix}$ **h)** $\begin{pmatrix} 0 & 2 \\ 3 & 7 \end{pmatrix}$

Problem 5. Let W be a subset of $\mathbb{R}^2$ that is a vector space using the addition and scalar multiplication operations from $\mathbb{R}^2$. Suppose that $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \in W$ and $\begin{pmatrix} -2 \\ 1 \end{pmatrix} \in W$. Prove that W is all of $\mathbb{R}^2$. (This is easy, as long as you remember that any vector space is closed under vector addition and scalar multiplication. This means that, given any vector is $\mathbb{R}^2$, you just need to be able to write that vector as a linear combination of the two given vectors.)

Problem 6. Remember that to show that a set is **not** a vector space, you only need to find one vector space property that fails, out of the ten properties that vector spaces must satisfy.

(a) Let S be the subset of $\mathbb{R}^2$ defined as $S = \{(x, y) \,|\, x + y = 1\}$. Show that S, using the addition and scalar multiplication operations from $\mathbb{R}^2$, is **not** a vector space.

(b) Let T be the subset of $\mathbb{R}^2$ defined as $T = \{(x, y) \,|\, x + y \geq 0\}$. Show that T, using the addition and scalar multiplication operations from $\mathbb{R}^2$, is **not** a vector space.