This homework is due by the end of the day on Wednesday, Sept. 9.
Be sure to show your work and explain your reasoning!

Problem 1. Two people solve a linear system of equations in two variables and they get the following sollution sets, where each set represents a line in $\mathbb{R}^2$:

$$A = \left\{ \begin{pmatrix} 3 \\ 2 \end{pmatrix} + a \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix} \ : \ a \in \mathbb{R} \right\} \qquad B = \left\{ \begin{pmatrix} 1 \\ -4 \end{pmatrix} + a \cdot \begin{pmatrix} 2 \\ 6 \end{pmatrix} \ : \ a \in \mathbb{R} \right\}$$

Can they both be correct? Explain why the two lines are actually the same line. First check that the point $(3, 2)$ is on both lines. Then explain why the two lines point in the same direction. And explain in words why all this shows that the two lines are the same.

Problem 2. Suppose two planes in $\mathbb{R}^3$ are given by the linear equations $x + y + z = 1$ and $Ax + By + Cz = D$. The intersection of the two planes can be empty, or it can be a line, or the planes could be identical. For each case, what has to be true about the constants A, B, C, and D in the second equation? Explain! (Hint: The intersection is the set of solutions to a system of two linear equations, and that set can be determined by putting the system into echelon form.)

Problem 3. Let $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_{n-1}$ be $n-1$ vectors in $\mathbb{R}^n$. Prove that there is a non-zero vector $\vec{x}$ in $\mathbb{R}^n$ that is orthogonal to $\vec{v}_i$ for all $i = 1, 2, \ldots, n-1$. (Hint: Think about linear equations! Write the condition as a linear system, and note that it is a homogeneous system.)

Problem 4. Let $\vec{v}_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$, and $\vec{v}_3 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$. Write the vector $\begin{pmatrix} 0 \\ 5 \\ -1 \end{pmatrix}$ as a linear combination of $\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$. To find the coefficients in the linear combination, set up a system of linear equations, and then solve that system.

Problem 5. Apply Gauss's method to put each matrix into echelon form. Based on your answer, state whether the matrix is singular or non-singular.

(a) $\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$ **(b)** $\begin{pmatrix} -1 & 0 & 1 & 0 \\ 3 & 2 & -2 & 4 \\ 2 & 1 & 0 & -1 \\ 1 & 1 & 0 & 1 \end{pmatrix}$