This homework on Sections 1.I.1 and 1.I.2 is due by the end of the day on Monday, Aug. 31. Be sure to show your work and explain your reasoning!

Note: Here is an example of applying row operations to a linear systtem:

$$\begin{array}{rrrcr} x & +3y & -2z & = & 1 \\ -2x & +y & -z & = & -3 \\ x & -y & +z & = & 2 \end{array} \quad \xrightarrow[-\rho_1+\rho_3]{2\rho_1+\rho_2} \quad \begin{array}{rrrcr} x & +3y & -2z & = & 1 \\ & 7y & -5z & = & -1 \\ & -4y & +3z & = & 1 \end{array}$$

$$\xrightarrow{\frac{4}{7}\rho_4+\rho_3} \quad \begin{array}{rrrcr} x & +3y & -2z & = & 1 \\ & 7y & -5z & = & -1 \\ & & \frac{1}{7}z & = & \frac{3}{7} \end{array}$$

Problem 1. The following systems of linear equations have unique solutions. Use Gauss's Method to put each system to echelon form, and then find the solution. As you apply row operations, show the result of each operation and show which operation you are applying. You can specify a row operation using the same notation as the textbook. You can combine several row operations of the form $\rho_j + k\rho_i$ into one step, as long as they use the same ρ_i.

(a)
$$\begin{array}{rrcr} 2x & -3y & = & -1 \\ x & +2y & = & 3 \end{array}$$

(b)
$$\begin{array}{rrrcr} & x_2 & +2x_3 & = & 3 \\ x_1 & -x_2 & -3x_3 & = & -2 \\ 2x_1 & +4x_2 & -x_3 & = & 0 \end{array}$$

(c)
$$\begin{array}{rrrcr} x & +y & +z & = & 1 \\ x & -y & -2z & = & 2 \\ 2x & +y & +z & = & 3 \\ x & -y & & = & 4 \end{array}$$

Problem 2. For each of the linear systems in problem **1**, rewrite the system in the form of an augmented matrix. For this short problem, *you do not need to show any work, just write the answers*. You just have to write the augmented matrix form of the original system of equations.

Problem 3. The following systems are already in echelon form. Each each system has an infinite number of solutions. Express the set of solutions in vector form. The answers will have a form similar to $\{\vec{v}_1 + a\vec{v}_2 + b\vec{v}_3 \mid a, b \in \mathbb{R}\}$, where v_1, v_2 and v_3 are column vectors of constants.

(a)
$$\begin{array}{rrrcr} x & -3y & -z & = & -1 \\ & 2y & +3z & = & 5 \end{array}$$

(b)
$$\begin{array}{rrrrrcr} 2x_1 & -x_2 & +3x_3 & +x_4 & -2x_5 & = & 3 \\ & & -x_3 & +2x_4 & -x_5 & = & 1 \\ & & & x_4 & +4x_5 & = & -2 \end{array}$$

Problem 4. The following augmented matrix represents a system of linear equations:

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 2 & 3 & -1 & 4 \\ 1 & 2 & -2 & b \end{array}\right)$$

For which values of the variable b, if any, does the system have exactly one solution? no solution? infinitely many solutions?