Math 135, Fall 2019, Test 2 Information

The second in-class test will take place on Friday, November 1. It will cover Chapters 4 through 10 in the textbook. Of course, you will also need to remember material about logic and sets from Chapters 1 and 2. Chapters 4 to 10 concentrate on general proof techniques, but they also include some new mathematical terms and concepts. As usual, the test will include definitions and other essay-type questions, as well as problems testing mathematical concepts such as divisibility, congruence modulo n, and rational and irrational numbers. There will also be several proofs, using different proof techniques, including at least one proof by induction. Given the time constraints of a one-hour test, the proofs should be fairly straightforward.

Proof techniques that you should know:

direct proof of a "$P \Rightarrow Q$" statement

proving a "for all" statement

proof by contrapositive

proof by contradiction

proof by cases

if-and-only-if proof

existence proof

proving $a \in A$, $A \subseteq B$, and $A = B$ for sets A and B

disproof by counterexample

proof by mathematical induction

proof by strong induction

Other terms and ideas that you should know for the test:

theorems, lemmas, corollaries

conjectures

even and odd numbers

divisibility for integers, $a \mid b$ if and only if $b = ka$ for some $k \in \mathbb{Z}$

prime numbers

there are infinitely many prime numbers

greatest common divisor, $\gcd(a, b)$

relatively prime integers, $\gcd(a, b) = 1$

congruence modulo n: $a \equiv b \pmod{n}$

rational number: can be written as $\frac{p}{q}$ where $p, q \in \mathbb{Z}$ and $q \neq 0$

irrational number: a real number that is not rational

$\sqrt{2}$ is irrational; $\sqrt{p}$ is irrational for any prime number p

π is irrational

base case of an induction; inductive case of an induction

inductive hypothesis

factorials:
$n! = n \cdot (n-1) \cdot (n-2) \cdots 2 \cdot 1$; $0! = 0$; $(n+1)! = (n+1) \cdot n!$ for $n > 0$

summation notation, $\sum_{i=1}^{n} a_i$

why proof by contradiction works

why disproof by counterexample works

why induction works; why the base case is necessary

how validity of induction follows from the well-ordering principle for $\mathbb{N}$

well-ordering principle for $\mathbb{N}$:
Every non-empty set of natural numbers has a smallest element

division algorithm:
If $a \in \mathbb{Z}$ and $b \in \mathbb{N}$, then $a = bq + r$ for some unique $q \in Z$ and $0 \leq r < b$.

if p is a prime number and $p \mid ab$, then $p \mid a$ or $p \mid b$

$\gcd(a, b)$ can be written as $\gcd(a, b) = ax + by$ for some integers x and y

every integer is congruent mod n to exactly one of $0, 1, \ldots, n-1$

Fundamental Theorem of Arithmetic:
Every integer $n \geq 2$ has a unique prime factorization