Math 135, Fall 2019, Test 1 Information

The first in-class test will take place on Friday, September 27. It will cover the following material: Chapters 1 and 2 in the textbook; Sections 1 to 3 from Chapter 4 in the textbook; and some basic ideas from the Topological Spaces handout. The questions on the test will include shorter and longer essay-type questions, problems about sets and logic (things like "if $A \subseteq B$, what can you say about $A \cap B$ and why" or "find the negation"), and simple proofs.

Some things that you should know about for the test:

sets; elements of a set; finite and infinite sets; notation $a \in A$, $a \notin A$

subset; notation $A \subseteq B$, $A \nsubseteq B$

the sets $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, and $\mathbb{R}$

the empty set, $\varnothing$; the empty set is a subset of every set

set notation such as $\{1, 2, 3\}$; set builder notation such as $\{n \in \mathbb{N} : n < 100\}$

cardinality of a finite set, $|A|$

operations on sets: union ($A \cup B$), intersection ($A \cap B$), set difference ($A \setminus B$)

ordered pais, such as $(1, 2)$, and cartesian product of sets ($A \times B$)

power set of a set ($\mathscr{P}(A)$)

universal set and the complement of a set in the universal set ($\overline{A}$)

indexed sets and the notations $\bigcap\limits_{\alpha \in I} A_i$ and $\bigcup\limits_{\alpha \in I} A_i$

mathematical logic; statements; open sentences

logical operations: AND ($P \wedge Q$), OR ($P \vee Q$), and NOT ($\sim P$)

the conditional operator ($P \Rightarrow Q$); if P then Q; P implies Q

the biconditional operator ($P \Leftrightarrow Q$); P if and only if Q

truth tables defining $P \wedge Q$, $P \vee Q$, $\sim P$, $P \Rightarrow Q$, $P \Leftrightarrow Q$

logical equivalence ($P \equiv Q$); using a truth table to prove logical equivalence

quantifiers; "for all" ($\forall$) and "there exists" ($\exists$)

notations like $\forall x \in A, P(x)$ and $\exists x \in A, P(x)$ and $\forall X \subseteq A, P(X)$

negations; negation of $P \Rightarrow Q$; negation of quantified expressions $\forall x, P(x)$ and $\exists x, P(x)$

translating logic to and from English

theorems and proofs; hypotheses and conclusion of a theorem

direct proof, for proving "if. . . then" theorems

even and odd integers

prime numbers

divisors, multiples, and the notation $a|b$ for "a divides b"

greatest common divisor $gcd(a, b)$ of two integers

least common multiple $lcm(a, b)$ of two integers

topological space; open sets in a topological space; closed sets in a topological space

Some "Laws" for Logic and Sets

Except for DeMorgan's laws, you are not required to know the names of these laws. Most of these laws are fairly obviously true, except for DeMorgan's and Distributive. In the "For sets" section, U represents a universal set. The "Identity" laws get their name from the fact that T and F in logic, and U and $\varnothing$ in set theory, have properties similar to 1 and 0 in arithmetic; 1 and 0 are called the "identities" for multiplication and addition. The names "Exclusded Middle," "Contradiction," and "Double Negation" are ordinarily used only for logic, but there are corresponding laws for sets.

Name	**For logic**	**For sets**
Commutative laws	$P \vee Q \equiv Q \vee P$ $P \wedge Q \equiv Q \wedge P$	$A \cup B = B \cup A$ $A \cap B = B \cap A$
Associative laws	$(P \vee Q) \vee R \equiv P \vee (Q \vee R)$ $(P \wedge Q) \wedge R \equiv P \wedge (Q \wedge R)$	$(A \cup B) \cup C = A \cup (B \cup C)$ $(A \cap B) \cap C = A \cap (B \cap C)$
Distributive laws	$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$ $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$	$A \cup (B \cap C) \equiv (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) \equiv (A \cap B) \cup (A \cap C)$
DeMorgan's laws	$\sim (P \vee Q) \equiv (\sim P) \wedge (\sim Q)$ $\sim (P \wedge Q) \equiv (\sim P) \vee (\sim Q)$	$\overline{A \cup B} = \overline{A} \cap \overline{B}$ $\overline{A \cap B} = \overline{A} \cup \overline{B}$
Idempotent laws	$P \vee P \equiv P$ $P \wedge P \equiv P$	$A \cup A = A$ $A \cap A = A$
Identity laws	$P \wedge T \equiv P, \quad P \wedge F \equiv F$ $P \vee T \equiv T, \quad P \vee F \equiv P$	$A \cap U = A, \quad A \cap \varnothing = \varnothing$ $A \cup U = U, \quad A \cup \varnothing = A$
Excluded Middle Contradiction	$P \vee (\sim P) \equiv T$ $P \wedge (\sim P) \equiv F$	$A \cup \overline{A} = U$ $A \cap \overline{A} = \varnothing$
Double Negation	$\sim (\sim P) \equiv P$	$\overline{\overline{A}} = A$
Additional facts for logic	$P \Rightarrow Q \equiv (\sim P) \vee Q$ $P \Leftrightarrow Q \equiv (P \Rightarrow Q) \wedge (Q \Rightarrow P)$ $P \Leftrightarrow Q \equiv (P \wedge Q) \vee ((\sim P) \wedge (\sim Q))$ $\sim (P \Rightarrow Q) \equiv P \wedge (\sim Q)$	$\sim (\forall x, P(x)) \equiv \exists x, \sim P(x)$ $\sim (\exists x, P(x)) \equiv \forall x, \sim P(x)$ $\forall x \in A, P(x) \equiv \forall x (x \in A \Rightarrow P(x))$ $\exists x \in A, P(x) \equiv \exists x (x \in A \wedge P(x))$

Some definitions...

A set A is a **subset** of a set B if every element of A is also an element of B.

In logical notation, $A \subseteq B$ if and only if $\forall x (x \in A \Rightarrow x \in B)$.

The **power set** of a set A is the set whose elements are all of the subsets of A.

In set notation, $\mathscr{P}(A) = \{S : S \subseteq A\}$.

$|A \times B| = |A| \cdot |B|, \qquad |\mathscr{P}(A)| = 2^{|A|}$.

The **converse** of $P \Rightarrow Q$ is $Q \Rightarrow P$,

The **contrapositive** of $P \Rightarrow Q$ is $(\sim Q) \Rightarrow (\sim P)$.

An integer a is **even** if there is an integer k such that $a = 2k$.

An integer a is **odd** if there is an integer k such that $a = 2k + 1$.

For integers a, b, we say $a|b$ (a "**divides**" b) if there is an integer k such that $b = ka$.

A natural number n is **prime** if $n > 1$ and its only positive divisors are 0 and n.

For integers a and b, $gcd(a, b)$ is the largest natural number that is a divisor of both a and b.

For integers a and b, $lcm(a, b)$ is the smallest natural number that is a multiple of a and b.