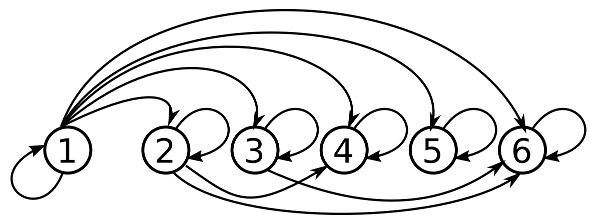
**Exercise 11.1.2** $R = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3)$ $(3, 6), (4, 4), (5, 5), (6, 6)\}$. As a diagram:




**Exercise 11.1.4** The set $A$ consists of all of the elements that appear as dots in the diagram and that can be related to each other by the relation, so $A = \{0, 1, 2, 3, 4, 5\}$. The relation itself is the set of ordered pairs $(a, b)$ where $a$ is a connected to $b$ by an arrow that starts at $a$ and ends at $b$. So $R = \{(0, 0), (1, 1), (2, 2), (3, 3)$ $(4, 4), (5, 5), (0, 4), (1, 3), (1, 5), (2, 4),$ $(3, 1), (4, 0), (4, 2), (5, 1)\}$.

**Exercise 11.1.10** $R$ is the relation $\neq$. That is $aRb$ if and only if $a \neq b$. As a set, the relation $R$ consists of all ordered pairs except for pairs $(x, x)$ where the second coordinate is equal to the first. After removing $\{(x, x) : x \in \mathbb{R}\}$, what's left is all ordered pairs $(x, y)$ where $x \neq y$.

**Exercise 11.2.2** The relation $R$ is not reflexive because $(a, a) \notin R$, that is $\sim(aRa)$. It is not symmetric because, for example, $(a, b) \in R$ but $(b, a) \notin R$, that is, $aRb$ but $\sim(bRa)$. It **is** transitive. To see that it is transitive, we have to look at all cases where $xRy$ and $yRz$ and check that $aRz$ as well. By inspection (or drawing a diagram), we see that $aRb$ and $bRc$ are true so we need $aRc$ to also be true, which it is. Similarly, $bRc$ and $cRb$ are true, so $bRb$ also needs to be true, and it is.

**Exercise 11.3.2** The two equivalence classes must be $\{b, c\}$ and $\{a, d, e\}$: Because $bRc$, $b$ and $c$ are in the same equivalence class. Because $aRd$ and $dRe$, $a$, $d$, and $e$ are in the same equivalence class. And we are given that there are two equivalence classes. So, as a set, $R = \{(b, b), (c, c), (b, c), (c, b), (a, a), (d, d), (e, e), (a, d), (d, a), (a, e), (e, a), (d, e), (e, d)\}$.

**Exercise 11.3.4** The facts that $aRd$ and $eRd$ already show that $a$, $d$, and $e$ are in the same equivalence class. The facts that $bRc$ and $cRe$ show that $b$, $c$, and $e$ are in the same equivalence class. Since $e$ is in both of those equivalence classes, they must in fact be the same equivalence class. So there is only one equivalence class, containing $a$, $b$, $c$, $d$, and $e$.

**Exercise 11.3.10** Let $R$ and $S$ be equivalence relations on a set $A$, and let $T = R \cap S$. This means that $aTb$ if and only if both $aRb$ and $aSb$. We show $T$ is an equivalence relation by showing that it is reflexive, symmetric, and transitive.

For $x \in A$, we know that $(x, x) \in R$ and $(x, x) \in S$. It follows that $(x, x)$ is in their intersection $T$. So $T$ is reflexive.

Suppose $(x, y) \in T$. Then by definition of intersection, $(x, y) \in R$ and $(x, y) \in S$. Since $R$ and $S$ are symmetric, we know that $(y, x) \in R$ and $(y, x) \in S$. Again by definition of intersection, $(y, x) \in T$. So $T$ is symmetric.

Suppose $(x, y)$ and $(y, z)$ are both in $T$. Then they are both in $R$ and both in $S$. Since $R$ and $S$ are transitive, we know that $(x, z)$ is in both $R$ and $S$, and it follows that $(x, z)$ is in $T$. So $T$ is transitive.

**Exercise 11.3.12** It is **not** true that the union of two equivalence relations is an equivalence relation. We prove this by giving a counterexample. Let $A = \{a, b, c\}$. Let $R$ be the relation on $A$ given by $R = \{(a, a), (b, b), (c, c), (a, b), (b, a)\}$ and let $S$ be the relation on $A$ given by $S = \{(a, a), (b, b), (c, c), (b, c), (c, b)\}$. $R$ and $S$ are equivalence relations. Note that the equivalence classes for $R$ are $\{a, b\}$ and $\{c\}$, while the equivalence classes for $S$ are $\{a\}$ and $\{b, c\}$. But $R \cup S$ is not an equivalence relation because $(a, b) \in R \cup S$ and $(b, c) \in R \cup S$ but $(a, c) \notin R \cup S$. That is, $R \cup S$ is not transitive. (Remark: $R \cup S$ **is** always reflexive and symmetric.)

**Exercise 11.4.2** There is one partition that has just one subset: $\{\{a, b, c\}\}$. There is one partition that has three subsets: $\{\{a\}, \{b\}, \{c\}\}$. There are three partitions that have two subsets. If there are two subsets, one of the subsets contains one element and the other contains two elements, so the partition is determined by which element is in a set by itself. The two-subset partitions are: $\{\{a\}, \{b, c\}\}$, $\{\{b\}, \{a, c\}\}$, $\{\{c\}, \{a, b\}\}$.

**Exercise 11.4.6** Two integers $n$ and $m$ are related if and only if they have the same absolute value. For example, $2\,P\,{-2}$ because $|2| = |-2|$, but $\sim(2\,P\,{-3})$ because $|2| \neq |-3|$.