Math 135, Fall 2019, Homework 6

This short homework is due next Wednesday, October 9, in class. Homework 5, your first LaTeX assignment, is due on Monday, October 7. For Homework 6, you should turn in a hard copy. You have the option of either writing your solutions by hand or processing them with LaTeX and printing the result, but in either case, you should turn in your work on paper.

1. Prove: If a and b are integers, then $(a + b)^3 \equiv a^3 + b^3 \pmod{3}$.

2. Prove using the contrapositive method: If the product of two integers is odd, then both of the numbers are odd.

3. Prove using proof by contradiction: If a is a rational number and b is an irrational number, then $a + b$ is an irrational number.

4. Prove using the contrapositive method: If n is an integer and $n \equiv 2 \pmod{3}$, then n is not a square. (Saying that n is not a square means that there is no integer a such that $n = a^2$.)

5. Prove using proof by contradiction: No rational number is a solution of the equation $x_x^3 + 1 = 0$. (Outline of proof: Suppose $x = \frac{p}{q}$ is a solution, where p and q are not both even. Substitute $\frac{p}{q}$ into the equation, and multiply by q^3 to clear the denominator. Now show that the left side of the equation is odd, which means that it cannot be zero. To show the left side is odd, use a proof by cases.)