Math 135, Fall 2019, Homework 4

This homework is due next Wednesday, September 25.
Reminder: There is a test next Friday, September 27.

1. Express the logical negation of each of the following sentences in natural, unambiguous English.

a) Math 135 is not meant to be fun.

b) The answer is greater than ten and less than twenty.

c) If Fred lives to be 100, he will still be a grouch.

2. Consider the statement, "If Casey strikes out, then there is no joy in Mudville." Express in natural English **a)** the *converse*, **b)** the *contrapositive*, and **c)** the *negation* of this statement.

3. Write the *negation* of each of the following statements in simplified form. In your answer, the *not* operator, $\sim$, should only be applied to simple terms, such as $\sim P(a)$ or $\sim L(x,y)$.

a) $\big(\forall x\,(P(x))\big) \vee \big(\forall x\,(Q(x))\big)$

b) $\forall x\,\big(P(x) \vee Q(x)\big)$

c) $\exists x\ \forall y\,\big(L(y,x) \to P(y)\big)$

d) $\forall n\,\big(\,Z(n) \to \exists k(Z(k) \wedge G(k,n))\,\big)$

4. Translate each of the following English sentences using predicates (things like "$P(x)$" or "$L(x,y)$") and quantifiers. Make up any predicates that you need. State what each predicate means. Try to capture as much of the meaning of the English statement as you can. For full credit, avoid using quantifiers in forms such as "$\forall x \in A$" and "$\exists x \in A$".

a) All cats are selfish.

b) Some politician is honest.

c) There is a book that no one has read.

d) Every integer is a real number.

e) There is a prime number that is greater than 1,000,000.

5. The sentence "Someone has the answer to every question" is ambiguous. Give two translations of this statement into logic using quantifiers and a predicate, and carefully explain the difference in meaning.

6. Prove: If n and m are odd integers, then $n+m$ is even.

7. Prove: If x and y are odd integers, then xy is odd.

8. Prove: For any integers a, b, and c, if $a|b$, then $a|bc$.

9. Prove: For any integers a, b, and c, if $a|b$ and $a|c$, then $a|(b+c)$.

10. Prove or disprove:

a) For any integer n, if n is divisible by 4, then n^2 is divisible by 4.

b) For any integer n, if n^2 is divisible by 4, then n is divisible by 4.