CPSC 229, Spring 2021 Sample Answers to Test #2

① Define the terms alphabet and language...

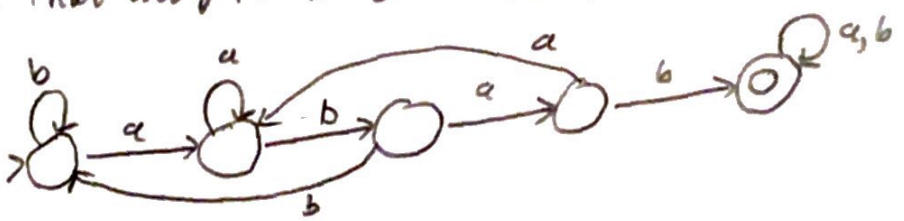
Answer: An alphabet, Σ, is a finite non-empty set. The elements of Σ are considered to be "symbols". A language over Σ is a set of strings made up of symbols from Σ.

② Let $L = \{a, ab, abb\}$. What strings are in the language L^*?

Answer: L^* contains the empty string plus all strings of a's and b's that start with a and do not contain bbb as a substring.

③ Draw a DFA that accepts $L = \{w \in \{a,b\}^* \mid w \text{ has substring } abab\}$

Answer:



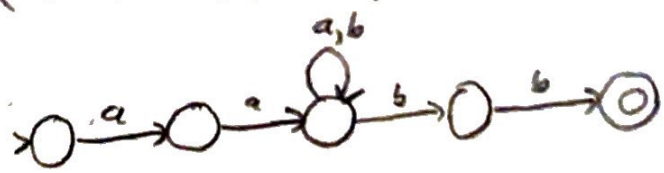
④ Suppose M is a DFA over the alphabet Σ. Explain what it means to say that a string $w \in \Sigma^*$ is accepted by M.

Answer: w is accepted by M if when M starts in its start state and reads the entire string, the state in which M finishes is an accepting state. (Formally, if $M = (Q, \Sigma, q_0, \delta, F)$, w is accepted if $\delta^*(q_0, w) \in F$.)

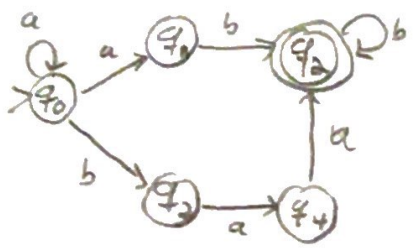
⑤ Consider the language $\{w \in \{a,b\}^* \mid w \text{ starts with aa and ends with bb}\}$. a) Give a regular expression for the language. b) Draw an NFA that accepts this language.

a) $aa(a|b)^*bb$

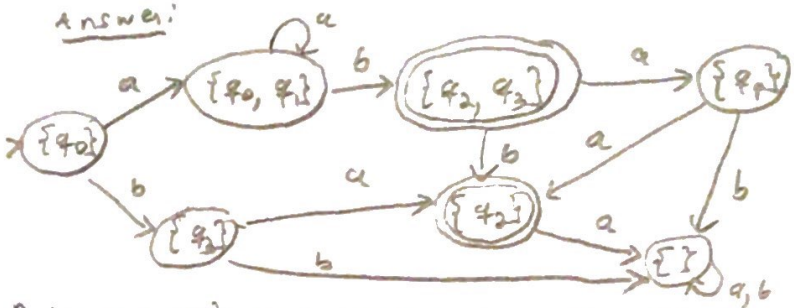
b)



⑥ Use the NFA-To-DFA conversion algorithm on this NFA:



Answer:



⑦ Find a CFG for $\{a^n b^m c^n \mid n, m \in \mathbb{N}\}$ and explain how it works.

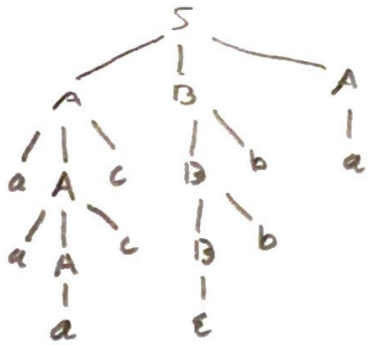
Answer: $S \to aSc$
$S \to T$
$T \to bT$
$T \to \varepsilon$

The first rule makes equal numbers of a's and c's, with all the a's on the left and c's on the right. Then T makes any number of b's, between the a's and c's. Using a new symbol, T, to make the b's ensures they can only be between the a's, c's.

⑧ Given this CFG, give a left derivation and parse tree for aaaccbba

$S \to ABA$
$A \to aAC$
$B \to Bb$
$A \to a$
$B \to \varepsilon$

Answer: $S \Rightarrow \underline{A}BA$
$\Rightarrow a\underline{A}cBA$
$\Rightarrow aa\underline{A}ccBA$
$\Rightarrow aaaccBA$
$\Rightarrow aaacc\underline{B}bA$
$\Rightarrow aaacc\underline{B}bbA$
$\Rightarrow aaaccbbA$
$\Rightarrow aaaccbba$



⑨ One of these languages is regular, one is context-free. Give a regular expression for the regular language and a CFG for the context-free.

a) $\{a^n b^m \mid n > m\}$

Answer: $S \to aSb$
$S \to aS$
$S \to \varepsilon$ (context-free)

b) $\{a^n b^m \mid n > 0 \text{ and } m > 1\}$

aa^*bbb^* (regular)

c) $\{a^n b^m \mid m = n^2\}$

(NOT context free)

⑩ a) What is nondeterminism? What is non-deterministic about NFA's?
b) What about grammars? Are they deterministic or non-deterministic?

Answer: a) Nondeterminism means that a computation can include steps where the action taken is not fully specified but can be selected, at random, from several possible actions. An NFA might have several different states it can jump to for a given input, and it can have ε-transitions that it can either follow or not. b) Grammars are non-deterministic since there can be several production rules that can be applied to the same non-terminal.