

This homework covers the reading from Chapter 3, Sections 1 and 2. It is due by the end of Wednesday, March 17, and will be accepted late with a 10% penalty until noon on Sunday, March 21.

1. [10 points] Consider the alphabet $\Sigma = \{a, b, c\}$. Let K , L , and M be the languages over M given by:

$$K = \{\varepsilon, a, b, c\} \quad L = \{aa, ab\} \quad M = \{c, cb, cbb, cbbb, cbbbbb, \dots\}$$

Find the following languages. In each case, specify the elements of the language using set notation, or give a clear description in words of the set of strings that make up the language. You do not have to justify your answers, but an explanation might get you some partial credit for an incorrect answer.

$$\begin{array}{lllll} \text{a) } K \cup L & \text{b) } L^2 & \text{c) } K^2 & \text{d) } M^2 & \text{e) } M^R \\ \text{f) } KL & \text{g) } LM & \text{h) } K^* & \text{i) } L^* & \text{j) } M^* \end{array}$$

2. [2 points] True or False, if true, give a proof; if false, give a counterexample: Let L be a language over an alphabet Σ . If $L^R = L$, then every string in L is a palindrome. (A palindrome is a string, x , such that $x^R = x$.)

3. [6 points] Let $\Sigma = \{a, b\}$. Consider the languages that are generated by the following regular expressions over Σ . Give clear descriptions in words of the set of strings in each language. You need to make it clear how the languages differ. (Note that the following is **not** an acceptable sort of answer: Any number of a's, then a b, then any number of a's, then a b, then any number of a's, then a b, then any number of a's. Give simple characterizations of the strings that are generated.)

$$\begin{array}{ll} \text{a) } a^*(bbb)a^* & \text{b) } (a|b)^*bbb(a|b)^* \\ \text{c) } a^*ba^*ba^*ba^* & \text{d) } a^*(b|\varepsilon)a^*(b|\varepsilon)a^*(b|\varepsilon)a^* \end{array}$$

4. [4 points] Let $\Sigma = \{a, b\}$. Consider the languages that are generated by the following regular expressions over Σ . Give clear descriptions in words of the set of strings in each language.

$$\text{a) } ab(a|b|c)^*ba \mid aba \quad \text{b) } (b|c|ab|ac)^*(a|\varepsilon)$$

5. [10 points] For each of the following languages, give a regular expression that generates that language. Justify your answers by explaining why the regular expression generates the strings of the language. Be careful to note the alphabet in each case, and be careful to account for **all** of the strings that satisfy the given condition.

$$\begin{array}{l} \text{a) } L_1 = \{x \in \{a, b\}^* \mid \text{the first and last characters in } x \text{ are different}\} \\ \text{b) } L_2 = \{x \in \{a, b\}^* \mid x \text{ the number of } b\text{'s in } x \text{ is an even number}\} \\ \text{c) } L_3 = \{x \in \{a, b, c\}^* \mid x \text{ contains at least one } c, \text{ and there are no } a\text{'s before that } c\} \\ \text{d) } L_4 = \{x \in \{a, b, c\}^* \mid x \text{ contains at least one of the substrings } aaa \text{ or } bbb\} \\ \text{e) } L_5 = \{x \in \{a, b, c\}^* \mid x \text{ contains both of the substrings } aaa \text{ and } bbb\} \end{array}$$