This homework covers the reading from Chapter 1, Sections 8 and 9, and Chapter 2, Section 1. It is due by the end of Wednesday, February 24, and will be accepted late with a penalty until noon on Saturday, February 27.

1. (*3 points*) Write out each of the following sums in full, without using summation notation. You are **not** being asked to compute the value of the sums, just to expand each sum into a normal sum of several terms!

a) $\displaystyle\sum_{n=1}^{5} n \cdot 5^n$ **b)** $\displaystyle\sum_{i=1}^{7} (2i-1)$ **c)** $\displaystyle\sum_{k=3}^{6} \frac{k}{k^2+1}$

2. (*3 points*) Use a proof by induction to show that for any integer $n \geq 1$, $\displaystyle\sum_{i=1}^{n} (2i-1) \;=\; n^2$

3. (*3 points*) Use a proof by induction to show that for any integer $n \geq 1$, $n^3 + 2n$ is a multiple of 3. (That is, there is some integer j such that $n^3 + 2n = 3j$.)

4. (*3 points*) Use a proof by induction to show that the following method correctly finds the sum of array elements `A[0], A[1], ..., A[N-1]` for all $N \geq 1$.

```
int recursive_sum( int[] A, int N ) {
    if ( N == 1 )
        return A[0];
    else {
        return A[N-1] + recursive_sum( A, N-1 );
    }
}
```

5. (*4 points*) Let $A = \{1,2,3,4,5,6,7,8,9\}$; let $B = \{2,4,6,8,10,12,14,16,18\}$; and let $C = \{n \in \mathbb{Z} \mid -5 \leq n \leq 5\}$. Find the following sets. (For this exercise, you do **not** need to justify your answers.)

a) $A \cup B$ **b)** $A \cap B$ **c)** $A \smallsetminus B$ **d)** $B \smallsetminus A$

e) $A \cap C$ **f)** $\mathbb{N} \cup C$ **g)** $\mathbb{N} \smallsetminus C$ **h)** $\mathbb{Z} \smallsetminus A$

(Recall that $\mathbb{N}$ is the set of natural numbers, $\mathbb{N} = \{0, 1, 2, 3 \dots\}$, and $\mathbb{Z}$ is the set of integers.)

6. (*2 points*) Let A be the set $A = \{\, \varnothing, a, \{a\} \,\}$. Write out the power set, $\mathscr{P}(A)$. (It has 8 elements. You do not have to justify your answer.)

7. (*3 points*) Let A be any set. What can you say about $A \cup A$? about $A \cap A$? about $A \smallsetminus A$? Justify your answer, either informally or by using the definitions of $\cup$, $\cap$, and $\smallsetminus$.

8. (*3 points*) Prove: If A, B, and C are any sets and $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cap C$.