This homework covers the reading from Chapter 1, Sections 6 and 7, along with a couple of leftover questions from Section 5. It is due by the end of Wednesday, February 17, and will be accepted late with a penalty until noon on Saturday, February 20.

Questions 1 is about formal proofs of validity, as in Section 1.5. For the remaining questions, you should give informal, but careful and complete, proofs of the kind that are typically given by mathematicians. In your proofs, you can use the following facts without proving them:

- *The sum of any two rational numbers is rational.*
- *The number $\sqrt{2}$ is irratinal.*
- *The number π is irrational.*

You can also use basic facts from algebra.

1. (*6 points*) Translate each of the following arguments, expressed in English, into formal logic, and determine whether the argument is valid. If the argument is valid, give a formla proof of validity. If it is not valid, show that it is not valid.

a) In order to get a B.S. degree, Joe must take a math class or a computer science class. If Joe doesn't understand algebra, Joe can't take a math class. Joe has a B.S. degree, but Joe doesn't understand algebra. So Joe must have taken a computer science class.

b) If Bill stays up late partying, he is tired the next day. If Bill is tired and there is a test, he doesnt do well on the test. If Bill does well on a test, he celebrates. There was a test today, and Bill is not celebrating, so he must have stayed up late partying last night.

2. (*3 points*) Prove that for any integer n, the number $n^2 + n$ is even. [Hint: Consider a proof by cases, looking at the case where n is even and the case where n is odd.]

3. (*3 points*) Suppose that x, y, and z are integers such that $x + y + z$ is greater than 30. Prove that at least one of x or y or z is greater than 10. [Hint: Consider a proof by contradiction.]

4. (*5 points*) Prove:

a) The product xy of any two rational numbers x and y is rational.

b) For any real number x, if x^2 is an irrational number, then x is also irrational. [Hint: Consider a proof by contrapositive.]

5. (*2 points*) Disprove: For any real number x, if x is an irrational number, then x^2 is also irrational.

6. (*3 points*) Prove: For any real number x, either x is irrational or $\pi - x$ is irrational.

7. (*3 points*) Prove: Let a, b, and c be integers, and assume that $a|b$ and that $b|c$. Then $a|c$.