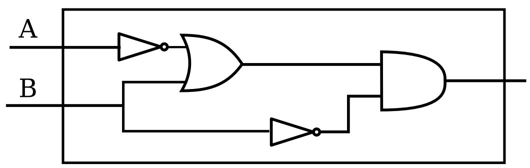
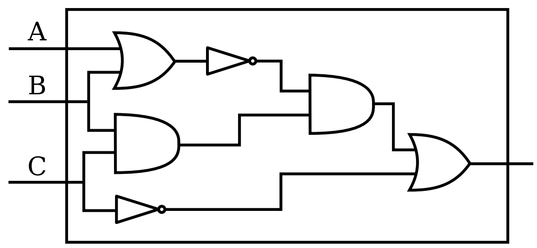
This homework covers the reading for the second week of classes: Chapter 1, Sections 3, 4, and 5. It is due by the end of the day on Wednesday, February 10. After that, it can be turned in up until Saturday, February 13, with a 10% penalty. You can work on these exercises with other people in the class, but you should write up your solutions in your own words to turn in. Remember that unsupported answers will receive little or no credit.

1. (*3 points*) Find the Boolean expression that gives the output of each circuit as a function of its inputs. (Show your work by redrawing the circuit and labeling the output of each logic gate.)

a)



b)



2. (*3 points*) Draw a logic circuit that computes each of the following Boolean expressions:

a) $(A \vee (\neg B)) \wedge (\neg(A \wedge (\neg B)))$

b) $(A \wedge (\neg B) \wedge C) \vee ((\neg A) \wedge B \wedge C)$

3. (*4 points*) Express the following statements in predicate logic. Try to express as much of the meaning as you can. Give the meaning of each predicate that you use. If it is not clear what the domain of discourse is for a predicate, state the domain of discourse explicitly.

a) Every elephant is gray.

b) There is a pink elephant.

c) Everyone owns a pink elephant.

d) There is a city where all elephants are pink.

4. (*2 points*) The following two Boolean expressions are not logically equivalent. Explain the difference in the meaning of the two expressions. Give an example of two specific predicates for which one of the expressions is true while the other one is false. (One possibility is two predicates for which the domain of discourse is integers.)

$$\big(\forall x, P(x)\big) \vee \big(\forall x, Q(x)\big) \qquad \text{and} \qquad \forall x\big(P(x) \vee Q(x)\big)$$

5. (*4 points*) Simplify each of the following expressions. Simplify the answer, so that the operator $\neg$ is only applied to individual predicates. (Show your work by writing a chain of logical equivalences, starting from the given expression.)

a) $\neg\big(\,\forall x\big(P(x) \vee Q(x)\big)\,\big)$

b) $\neg\big(\,\forall x(P(x) \rightarrow (Q(x) \wedge R(x)))\,\big)$

c) $\neg\big(\,\exists y(H(y) \wedge \forall x L(x,y))\,\big)$

d) $\neg\big(\,\forall x\,\exists y\,\exists z\,(L(x,y) \wedge G(x,z))\,\big)$

6. (*3 points*) Use a *truth table* to show that the following argument is valid. And then, explain in English why it *makes sense* that this argument is valid. (What does the argument *mean*?)

$$\begin{array}{l} p \rightarrow q \\ (\neg p) \rightarrow q \\ \hline \therefore\ q \end{array}$$

7. (*6 points*) Give a formal proof for each of the following valid arguments. For each step in the proof, give the justification for that step.

a)
$$\begin{array}{l} p \rightarrow q \\ q \rightarrow (r \vee s) \\ \neg s \\ p \\ \hline \therefore\ r \end{array}$$

b)
$$\begin{array}{l} (p \wedge q) \rightarrow (r \vee s) \\ \neg r \\ p \rightarrow q \\ p \\ \hline \therefore\ s \end{array}$$

c)
$$\begin{array}{l} p \rightarrow r \\ (r \wedge s) \rightarrow t \\ q \rightarrow \neg t \\ s \\ q \\ \hline \therefore\ \neg p \end{array}$$