      This homework covers the reading for the first week of classes: Chapter 1, Sections 1 and 2 (but mostly Section 1). Rembember that you can discuss these exercises with other people in the class, but you should write up your own solutions in your own words to turn in. Remember that unsupported answers will receive little or no credit.

      This homework is due by the end of the day on Wednesday, February 3. However, since it is the first homework of the semester, it will be accepted a day or two late without penalty. All homework must be submitted through Canvas in PDF form.

1. (*4 points*) Draw truth tables to prove the following logical equivalences (and justify your answer by saying what it is about the table that proves equivalence):

   a) $p \vee (q \vee p) \equiv p \vee q$

   b) $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$

2. (*4 points*) Now, prove the same logical equivalences using Boolean Algebra. That is, find a chain of logical equivalences leading from the left side of the equivalence to the right side, where a single definition or rule of Boolean algebra is applied in each step. For each step, state what definition or rule you are applying. (One rule that you will need is the fact that $a \rightarrow b \equiv (\neg a) \vee b$.)

   a) $p \vee (q \vee p) \equiv p \vee q$

   b) $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$

3. (*2 points*) Convert each of the following English statements into propositional logic. You should introduce symbols (such as p, q, d, f, etc.) to stand for the simple propositions that occur in the statements. State clearly what each symbol stands for. Try to express as much of the meaning of the sentence as possible.

   a) *Jack is smart but not lucky.*

   b) *If I have a choice, then I don't eat broccoli.*

4. (*3 points*) Consider the English statement, "*Achilles is brave and famous or long-lived.*" This statement is ambiguous. Give two translations into propositional logic. Then, for each translation into logic, give an unambiguous English statement with the same meaning.

5. (*3 points*) Express the negation of each of the following sentences in natural, unambiguous English.

   a) *The answer is greater than ten and is less than or equal to 20.*

   b) *Jack is smart but not lucky.*

   c) *If Cassandra tells the truth, she will be believed.*

6. (*4 points*)

   a) Give (**1**) the *converse*, (**2**) the *contrapositive*, and (**3**) the *negation* of the proposition:
   $$p \rightarrow (\neg q)$$

   b) Consider the statement, "*If Casey strikes out, then there is no joy in Mudville.*" Express in natural English (**1**) the *converse*, (**2**) the *contrapositive*, and (**3**) the *negation* of this statement.

7. (*5 points*) Consider an ordinary poker deck of 52 playing cards. Using the definitions of the logical operators, determine the number of cards in the deck for which each of the following statements is logically true. Don't forget to justify each answer! The last two parts are tricky; you need to remember the logical meaning of $p \rightarrow q$.

   a) "This card is both an Ace and a Spade"?

   b) "This card is either an Ace or a Spade"?

   c) "This card is an Ace but is not a Spade"?

   d) "If this card is a Ace, then it is a Spade"?

   e) "If this card is an Ace, then it is a King"?